MATH266: First midterm exam, February 14th

Name: _____

1. Find the solution of the IVP
$$y' = \frac{x^2 + 1}{y}, \quad y(1) = \sqrt{3}.$$

2. Find the general solution to
$$xy' + y = \sqrt{x}, \quad x > 0.$$

3. Find the general solution to
$$xy' = y + 2\sqrt{xy}, \quad x, y > 0.$$

4. Uranium-238 has a half-life 4.5×10^9 years. How long will it take 500 gram of Uranium-238 to decay to 400 gram? (Recall that the radioactive substances decay at a rate proportional to the amount.) To obtain an approximate answer, you can take $\log \frac{5}{4} / \log 2 \approx 0.32$.

5. Find and classify equilibria of the autonomous differential equation

$$\dot{x} = 4x - x^3.$$

Draw the phase portrait and sketch several integral curves of this equation.